Table 1. Values of B_3 which makes the heat loss in case of no-fin larger than that in the case of a fin of infinite length for given values of B_1 and B_2

	B_3 for		
B_2/B_1	$B_1 = 0.01$	$B_1 = 0.1$	$B_1 = 1$
0	0.0704	0.2162	0.5854
0.25	0.0787	0.2438	0.6934
0.5	0.0863	0.2678	0.7702
0.75	0.0932	0.2893	0.8301
1	0.0996	0.3087	0.8796

(a) if L for $0.99(Q/k\theta_0)_{\max}$ decreases monotonically as the ratio of B_2/B_1 increases, then the fin is useful for all given values of B_1 and B_2;

(b) if L for $0.99(Q/k\theta_0)_{\max}$ varies irregularly as the ratio of B_2/B_1 increases, then a check using Table 1 must be made to determine the fin's usefulness;

(c) if L for $0.99(Q/k\theta_0)_{\max}$ is nearly zero as the ratio of B_2/B_1 increases, then the fin is not useful for all values of B_1 and B_2.

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Diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface

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INTRODUCTION

IN THIS paper, the effect of thermal-diffusion and diffusion-thermo on transient and steady natural convection heat and mass transfer from a vertical surface are investigated numerically. A helium–air mixture was selected as the fluid pair used in the study due to its radically different thermodynamic properties as compared to other fluid pairs. Results showing steady temperature and concentration distributions and the total heat and mass transport from the wall with and without heat and mass transfer coupling are presented. Also, the transient variation of the heat flux from the wall including and neglecting the coupling effects are documented.

The effect of diffusion-thermo and thermal-diffusion on the transport of heat and mass were developed from the kinetic theory of gases by Chapman and Cowling [1]. Hirshfelder *et al.* [2] explained the phenomena and derived the necessary formulae to calculate the thermal-diffusion coefficient and the thermal-diffusion factor for monatomic gases. Although the derivation is restricted to monatomic gases, they found that the error involved with applying the formulae to polyatomic gas mixtures is small.

Hall [3] developed the energy, diffusion and momentum equations for multicomponent systems. He further simplified the equations of motion, energy and diffusion for a steady compressible, boundary layer flow of a binary mixture over a flat plate.

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NOMENCLATURE

c_i mass fraction
C_p constant pressure specific heat
D_{12} coefficient of binary diffusion
D_T coefficient of thermal-diffusion
g gravitational acceleration
Gr Grashof number for heat transfer, defined by equation (14)
Gr_{eff} effective Grashof number for combined heat and mass transfer, defined by equation (13)
Gr_m Grashof number for mass transfer, defined by equation (15)
J mass flux
k thermal conductivity
k_T thermal-diffusion ratio, D_T/D_{12}
L length of the plate
M molecular weight
n mole fraction
Pr Prandtl number
q heat flux
Q nondimensional wall heat flux, defined by equation (12)
Q_0 steady-state nondimensional wall heat flux for heat transfer only, 0.3711
R specific gas constant
R_{Gr} Grashof number ratio, Gr_m/Gr
Sc Schmidt number
t time
T absolute temperature
T_r nondimensional temperature ratio, defined by equation (9)
u velocity parallel to the plate
v velocity perpendicular to the plate
x coordinate along the plate
y coordinate perpendicular to the plate.

Greek symbols
α_T thermal-diffusion factor, defined by equation (7)
β coefficient of thermal expansion
η nondimensional position, $(y/L)(L/x)^{1/4}(Gr_{eff,L})^{1/4}$
θ nondimensional temperature
ν kinematic viscosity
ρ density
τ nondimensional time, $(t\nu/L^2)(L/x)^{1/2}(Gr_{eff,L})^{1/2}$.

Subscipts
1 diffusing gas
2 inert gas
∞ free stream
L length of the plate
w wall
x x direction
y y direction.

Superscript
$\bar{}$ dimensionless quantities.

Baron [4, 5] analyzed steady flow of helium–air and Freon 13–air mixtures. He found that the heat flux and recovery temperature changed considerably if the diffusion-thermo and thermal-diffusion effects were neglected. He also compared his results with experimental data and found that the experimental data agree fairly well with the analytical results that included the heat and mass transfer coupling effect.

Tewfik [6] studied the steady flow of binary mixtures of hydrogen–helium and carbon dioxide–air over a flat plate. He found that the Nusselt number, including thermodynamic coupling, could be up to 3% different from the case where coupling was neglected. He also found that the effects of hydrogen and helium injection are larger than in the case of carbon dioxide injection.

Sparrow *et al.* [7] numerically investigated plate, axisymmetric stagnation flow and planar, free convection stagnation flow including and neglecting diffusion-thermo and thermal-diffusion effects. Binary mixtures of various gases with air were included in the study and the relative importance of the diffusion-thermo and thermal-diffusion effects were discussed.

Mills and Wortman [8] analyzed steady plane stagnation flow for various binary air mixtures. They observed the effect of heat and mass transfer coupling for hydrogen injection to the boundary layer and concluded that, for this case, the effect of coupling is important. Kendall and Bartlett [9] studied steady flow over a flat plate. They found, for this case, the thermal-diffusion effect was small, since the temperature gradients were small.

Sparrow *et al.* [10] also studied the case of natural convection heat and mass transfer from a horizontal cylinder for a helium–air mixture. They found that for the helium–air mixture, the coupling is very important and for increased Gr_m the heat flux can change direction and heat would flow to the wall, although the free stream temperature was lower than the wall temperature.

GOVERNING EQUATIONS

Applying the Boussinesq approximation and assuming each component behaves like an ideal gas, the governing equations for combined natural convection heat and mass transfer from a vertical surface are similar to those presented by Hall [3]. In nondimensional form they are continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

momentum equation

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\frac{Gr_m}{Gr}\bar{c}_1+\theta+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \tag{2}$$

diffusion equation

$$\frac{\partial \bar{c}_1}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{c}_1}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{c}_1}{\partial \bar{y}}=\frac{1}{Sc}\left(\frac{\partial^2 \bar{c}_1}{\partial \bar{y}^2}+(1-\bar{c}_1c_{1w})\bar{c}_1\frac{\alpha_T}{T_r}\frac{\partial^2\theta}{\partial \bar{y}^2}\right.$$
$$\left.-(1-\bar{c}_1c_{1w})\bar{c}_1\frac{\alpha_T}{T_r^2}\left(\frac{\partial\theta}{\partial \bar{y}}\right)^2+(1-2\bar{c}_1c_{1w})\frac{\alpha_T}{T_r}\frac{\partial\theta}{\partial \bar{y}}\frac{\partial \bar{c}_1}{\partial \bar{y}}\right) \tag{3}$$

and the energy equation,

$$\frac{\partial\theta}{\partial \bar{t}}+\bar{u}\frac{\partial\theta}{\partial \bar{x}}+\bar{v}\frac{\partial\theta}{\partial \bar{y}}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial \bar{y}^2}+\frac{R}{C_p}\frac{\alpha_T}{Sc}$$
$$\times\frac{M_1M_2}{(M_1(1-\bar{c}_1c_{1w})+M_2\bar{c}_1c_{1w})^2}\frac{M_2}{M_2-M_1}\frac{Gr_m}{Gr}$$
$$\times\left\{\frac{\partial^2 \bar{c}_1}{\partial \bar{y}^2}+\frac{\alpha_T(1-c_{1w}\bar{c}_1)\bar{c}_1}{T_r}\frac{\partial^2\theta}{\partial \bar{y}^2}\right.$$
$$+\frac{\alpha_T(1-2\bar{c}_1c_{1w})}{T_r}\frac{\partial\theta}{\partial \bar{y}}\frac{\partial \bar{c}_1}{\partial \bar{y}}$$

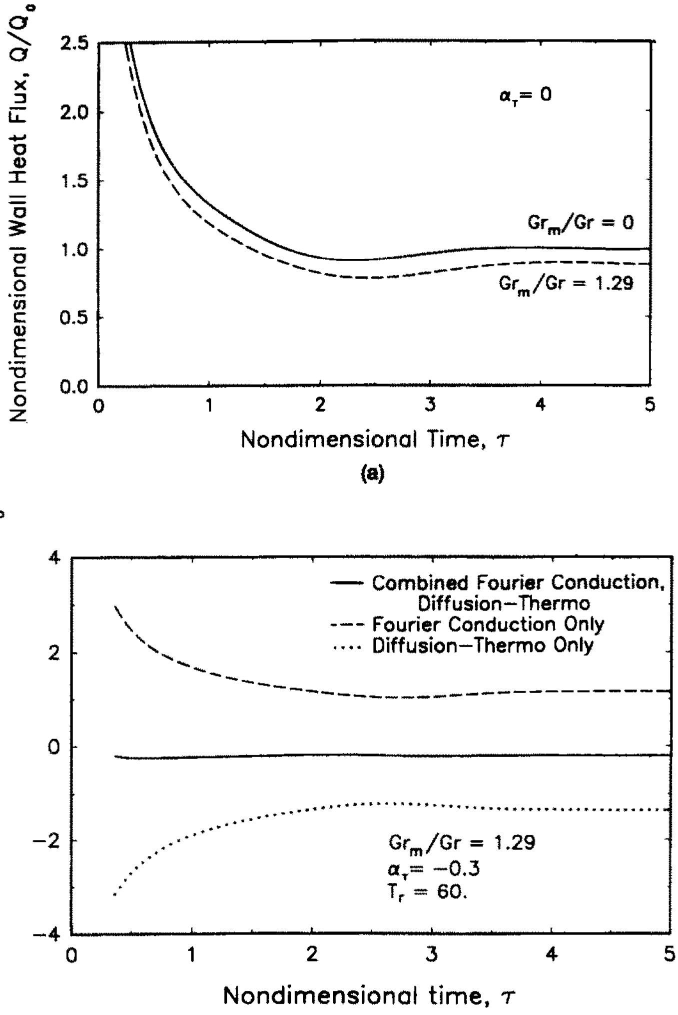

FIG. 1. (a) Transient variation of nondimensional wall heat flux with and without mass transfer. (b) Contribution of various terms on the transient nondimensional wall heat flux.

$$-\frac{2}{T_r}\,\frac{M_2}{M_1(1-\bar{c}_1c_{1w})+M_2\bar{c}_1c_{1w}}\,\frac{Gr_m}{Gr}\times\left[\left(\frac{\partial\bar{c}_1}{\partial\bar{y}}\right)^2+\frac{\alpha_T(1-c_{1w}\bar{c}_1)\bar{c}_1}{T_r}\,\frac{\partial\theta}{\partial\bar{y}}\,\frac{\partial\bar{c}_1}{\partial\bar{y}}\right]\Bigg\}$$
$$+\frac{1}{T_rSc}\,\frac{Gr_m}{Gr}\,\frac{M_2}{M_2-M_1}\,\frac{(C_{p1}-C_{p2})}{C_p}\times\left(\frac{\partial\theta}{\partial\bar{y}}\,\frac{\partial\bar{c}_1}{\partial\bar{y}}+\alpha_T\frac{(1-c_{1w}\bar{c}_1)\bar{c}_1}{T_r}\left(\frac{\partial\theta}{\partial\bar{y}}\right)^2\right) \quad (4)$$

where the nondimensionalized variables are defined as

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\quad \bar{c}_1=\frac{c_1}{c_{1w}}$$
$$\bar{u}=\frac{uL}{\nu}Gr^{-1/3}\quad \bar{v}=\frac{vL}{\nu}Gr^{-1/3}$$
$$\bar{t}=\frac{t\nu}{L^2}Gr^{2/3}\quad \bar{x}=\frac{x}{L}Gr^{1/3}\quad \bar{y}=\frac{y}{L}Gr^{1/3} \quad (5)$$

and these equations are subject to the following nondimensional boundary and initial conditions:

$$\begin{aligned}&\text{at } \bar{y}=0;\quad \bar{c}_1(\bar{t},\bar{x})=1,\quad \theta(\bar{t},\bar{x})=1,\\ &\qquad \bar{u}(\bar{t},\bar{x})=0,\quad \bar{v}(\bar{t},\bar{x})=\bar{v}_w,\\ &\text{as } \bar{y}\to\infty;\quad \bar{c}_1(\bar{t},\bar{x})=0,\quad \theta(\bar{t},\bar{x})=0,\\ &\qquad \bar{u}(\bar{t},\bar{x})=0,\\ &\text{at } \bar{t}=0;\quad \bar{c}_1(\bar{x},\bar{y})=0,\quad \theta(\bar{x},\bar{y})=0,\\ &\qquad \bar{u}(\bar{x},\bar{y})=0,\quad \bar{v}(\bar{x},\bar{y})=0.\end{aligned} \quad (6)$$

In these equations, the diffusion-thermo and thermal-diffusion effects give rise to two additional properties. These are the thermal diffusion factor, α_T, and the nondimensional temperature ratio, T_r. The thermal diffusion factor, which is independent of the concentration, is given by

$$\alpha_T=\frac{k_T}{n_1n_2} \quad (7)$$

where k_T, the thermal-diffusion ratio, which has a theoretical limit of 1.0 [2], is defined as

$$k_T=\frac{D_T}{D_{12}}. \quad (8)$$

The nondimensional temperature ratio is defined as

$$T_r \equiv \frac{T}{T_w - T_\infty}. \tag{9}$$

The heat flux at the wall, q_w, defined as

$$q_w = -k\frac{\partial T}{\partial y} + \frac{RT}{c_1 c_2} J_1 \tag{10}$$

where

$$J_1 = -\rho D_{12}\left[\frac{\partial c_1}{\partial y} + k_T \frac{\partial \ln T}{\partial y}\right] \tag{11}$$

was nondimensionalized in a manner similar to that used by Hellums and Churchill [11]. This method of nondimensionalization removes the dependence of the wall heat flux on position and renders the nondimensional wall heat flux (for heat transfer only) to be a function of Prandtl number only. The nondimensional heat flux is given by the following relation:

$$Q = \frac{q_w L}{k(T_w - T_\infty)}\left(\frac{x}{L}\right)^{1/4} Gr_{\text{eff}}^{-1/4} \tag{12}$$

where the effective Grashof number for combined heat and mass transfer, Gr_{eff}, is defined as given by refs. [12, 13] as

$$Gr_{\text{eff}} = \left(Gr + \left(\frac{Pr}{Sc}\right)^{1/2} Gr_m\right) \tag{13}$$

where

$$Gr = \frac{g\beta_\infty (T_w - T_\infty) L^3}{\nu^2} \tag{14}$$

and

$$Gr_m = \frac{g\left(\frac{M_2 - M_1}{M_1}\right) c_{1w} L^3}{\nu^2}. \tag{15}$$

RESULTS

Numerical simulations were conducted to investigate the effect of thermal-diffusion and diffusion-thermo on the heat and mass transfer from a vertical surface. All thermophysical properties were calculated assuming the mixture was primarily air and the properties of air were evaluated at the wall temperature, which was selected to be 100 K. This is reasonable, since the maximum concentration of He was less than 1% by weight and the largest temperature difference, between the wall and the free stream, was selected to be 5 K. This corresponds to the smallest value of T_r which, as shown in the governing equations, maximizes the heat and mass transfer coupling effect.

The equations were solved using an explicit finite-difference method. Results of the case that considered heat transfer only, without thermal-diffusion and diffusion-thermo, were compared to similarity solutions and to the numerical results given by Hellums and Churchill [11] using different grid sizes. For a grid having 20 nodes in the x direction and 40 nodes in the y direction, the temperature and velocity profiles and Nusselt number obtained from the present numerical solution agreed to within 0.1% with the results given by the similarity solution and Hellums and Churchill's numerical solution [11] for a range of Grashof numbers less than 10^6, well within the laminar range. This grid was used for all subsequent simulations.

Calculations were conducted neglecting either the diffusion-thermo or thermal-diffusion effect. The results were invariant whether or not the thermal-diffusion effect was included in the formulation. This is consistent with the results reported by Sparrow *et al.* [10].

Figure 1(a) shows the time-dependent variation of the heat transfer with and without mass transfer not including the effects of diffusion-thermo and thermal-diffusion. Time was nondimensionalized in a manner similar to Hellums and Churchill [11]; however, for the case of combined heat and mass transfer the effective Grashof number was used. The heat flux is normalized using Hellums and Churchill's value for the steady state heat flux from a wall for a case that includes only heat transfer effects (i.e. $Q_0 = 0.3711$). These results show that by defining the effective Grashof number, the case of heat transfer only and combined heat and mass transfer, neglecting heat and mass transfer coupling effects, have a similar shape. Also, for a nondimensional time of 5, the case of heat transfer only approaches Hellums and Churchill's [11] results. This effective Grashof number is then used to present all subsequent results.

In Fig. 1(b), the time dependence of the Fourier conduction and the diffusion-thermo effect are presented for the case of a helium–air mixture when heat and mass transfer coupling effects are included. These results show that the Fourier conduction and the diffusion-thermo effect are opposite in sign. This renders the heat flux at the wall, for this case, to be essentially time invariant for $\tau > 0.4$. Also, these results show that the direction of heat flow can be in the opposite direction to the normal flow of heat given by the temperature gradient. This effect was previously observed by Sparrow *et al.* [7, 10].

The effect of thermal-diffusion and diffusion-thermo on the steady temperature and concentration distributions was investigated next. These results for a helium–air mixture, given in Figs. 2(a) and (b), show that thermal-diffusion and diffusion-thermo do not affect the concentration field. However, the temperature field is appreciably affected. When thermal-diffusion and diffusion-thermo effects are introduced the temperature gradient at the wall is larger; however, the energy flux at the wall becomes negative, causing temperatures in the boundary layer to fall below the free stream temperature. This occurs because the contribution of the diffusion-thermo effect is in the opposite direction to the Fourier conduction effect. Increasing the ratio of Grashof numbers, R_{Gr}, increases the diffusion-thermo effect. The dimensionless energy equation, equation (4), shows that the dominant term of the diffusion-thermo effect, the concentration gradient term, is multiplied by R_{Gr}. Increasing this ratio increases the diffusion-thermo effect, resulting in a negative wall flux for sufficiently large Grashof number ratios. The last term in the energy equation is not directly related to the diffusion-thermo effect. It is due to the difference between the specific heats of the diffusing and inert species. This term is multiplied by the inverse of the nondimensional temperature ratio, T_r, and R_{Gr}. When T_r decreases, this term becomes significant. Since the effect of this term is in the same direction as the diffusion-thermo effect, for the helium–air mixture, reducing the nondimensional temperature ratio results in a negative heat flux at lower Grashof number ratios with smaller values of T_r, as shown in Fig. 3.

CONCLUSIONS

The effects of including thermal-diffusion and diffusion-thermo transfer mechanisms on the transport of heat and mass from a vertical wall are presented. Transient results of the combined heat and mass transfer problem are similar to the results when only heat transfer effects are included. In both cases, the nondimensional wall heat flux reaches a minimum at a nondimensional time of approximately 2.5 and steady-state is achieved when the nondimensional time is greater than 5.

For all cases considered in this study, the diffusion-thermo is the dominant effect and the omission of the thermal-diffusion effect in the formulation does not alter the results. Steady-state results show that the nondimensional wall heat flux decreases with increasing dimensionless temperature

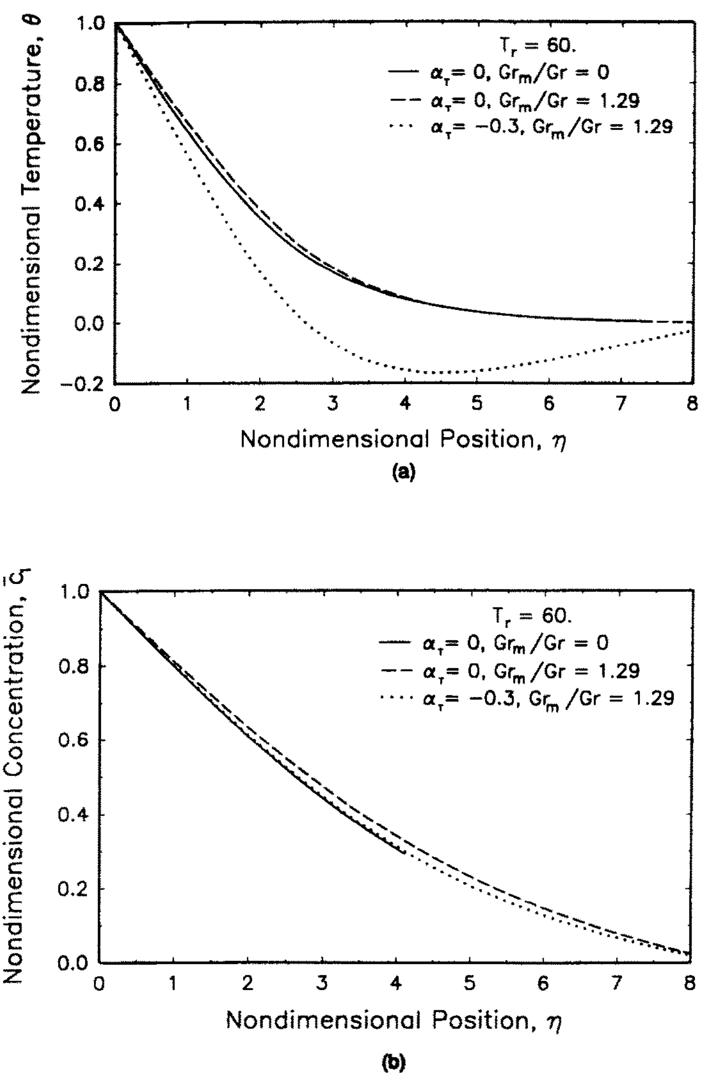

FIG. 2. (a) Effect of diffusion-thermo and thermal-diffusion on the nondimensional temperature profile. (b) Effect of diffusion-thermo and thermal-diffusion on the nondimensional concentration profile.

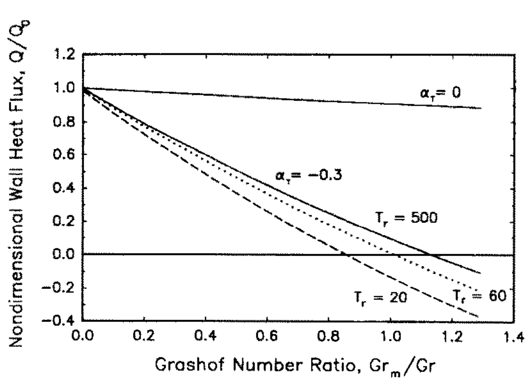

FIG. 3. Effect of Grashof number ratio and reduced temperature on the nondimensional wall heat flux.

ratio, indicating the strong effect of the dimensionless temperature ratio on the diffusion-thermo effect. Also, the nondimensional wall heat flux decreases slightly with increasing Grashof number ratio when no coupling is considered. However, as the dimensionless temperature ratio is decreased, the nondimensional wall heat flux shows a larger dependence on the Grashof number ratio. For a low value of the dimensionless temperature ratio, which is characteristic of low absolute temperature, the heat flux decreases with increasing Grashof number ratio and eventually reverses direction, causing heat to flow from the colder fluid to the warmer wall. In this case, the temperature in the boundary layer can fall below the free stream temperature. Despite the larger temperature gradient caused by the lower temperature in the boundary layer, the total wall heat flux decreases. This occurs because the diffusion-thermo effect is larger than the Fourier conduction effect, which renders the nondimensional wall heat flux to be negative (i.e. heat flow to the wall).

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